Q1

1

Similar triangles have three equal angles.
Use circle theorems to identify any corresponding angles that are equal between triangles *ABE* and *DCE*.

The same segment circle theorem states that angles at the circumference subtended by the same arc are equal.

Vertically opposite angles at a point are equal.

The statement that if three corresponding angles between two triangles are equal then the two triangles are similar must be made.

**Angle *ABE* = angle *DCA* as they are angles in the same segment.**
**Angle *BAE* = angle *CDE* as they are angles in the same segment.**
**Angle *AEB* = angle *DEC* as they are vertically opposite angles.**
*At least one of the three statements given, with reason* [1]
*At least two of the three statements given, with reasons* [1]
**All three angles are the same so the triangles are similar.**
*All three statements and explanation of similar triangles* [1]

Q2

The piece of card is mathematically similar to the photo so there will exist a scale factor which can be used to find the length of one of the shapes, given the length of the other.

Let the scale factor be *k*.

Form an equation using the two known sides, these are the width of the photo and the width of the card.

$$Width(card) = kWidth(photo)$$
$$15 = 10k$$

Solve to find $k$.

$$10k = 15$$
$$k = \frac{15}{10} = 1.5$$

[1]

Form an equation for the length of the photo and the length of the card.

$$Length(card) = kLength(photo)$$
$$l = 16k$$

Substitute $k = 1.5$ to find the length of the card.

$$l = 16(1.5)$$
$$l = 24$$

[1]

This is the length of the card after $x$ cm has been cut from the original length of 30 cm.

$$x = 30 - l$$
$$x = 30 - 24$$

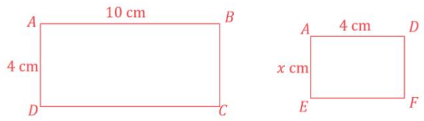
**$x = 6$ cm** [1]

Q3

The two rectangles are mathematically similar so there will exist a scale factor which can be used to find the length of one of the rectangles, given the length of the other.

Let the scale factor be *k*.

It will help to draw the two rectangles out side by side, with the length and the width of each in the same direction.




Form an equation using the two known sides, these are the two lengths (longer sides).

$$Length(ABCD) = k(Length(ADEF))$$
$$10 = 4k$$

Solve to find $k$.

$$4k = 10$$
$$k = \frac{10}{4}$$

[1]

Form an equation for the width of the two rectangles.

$$Width(ABCD) = k(Width(ADEF))$$
$$4 = kx$$

Substitute $k = \frac{10}{4}$ to find the width of the smaller rectangle

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Substitute $k = \frac{10}{4}$ to find the width of the smaller rectangle

$$4 = \frac{10}{4}x$$
$$10x = 16$$
$$x = \frac{16}{10} = 1.6$$

Find the area of rectangle *DAEF*.

$$Area = length \times width$$
$$= 4 \times 1.6$$

[1]

**Area = 6.4 cm$^2$** [1]

Q4

Calculate the area of the trapezium *QRST*, using the formula: $A = \frac{1}{2}(a+b)h$, where $a$ and $b$ are the lengths of the parallel lines and $h$ is the perpendicular height.

$$A = \frac{1}{2}(4+12) \times 10$$

[1]

**$80\ \text{cm}^2$** [1]

4b

Triangles *PRS* and *PQT* are similar.

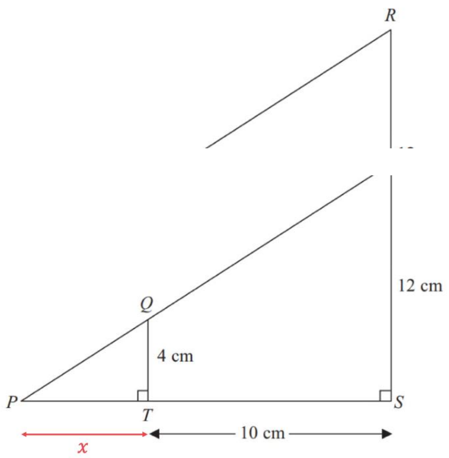
Compare the length of *RS* and *QT* to find the length scale factor (LSF).

$$LSF = \frac{12}{4} = 3$$

[1]

Divide the length *PS* by the LSF to find the length *PT*.

Let *PT* = $x$, so *PS* = $x + 10$




$$\frac{(x+10)}{3} = x$$

[1]

Multiply both sides by 3.

$$x + 10 = 3x$$

Subtract 10 from both sides.

$$10 = 2x$$

Divide both sides by 2.

$$5 = x$$

**5 cm** [1]

Q5

"*PTQ* is the diameter", so *TP* and *TQ* are two parts of the diameter. We know *TQ* and we can find *TP* using the **intersecting chord theorem,** $TP \times TQ = TR \times TS$

$$TP \times 3 = 12 \times 4$$

Find *TP* by dividing both sides by 3

$$TP = \frac{12 \times 4}{3} = 16$$

[1]

Now find the diameter by adding *TP* and *TQ*

$$\text{diameter} = TP + TQ = 16 + 3 = 19$$

Finally, find the radius by dividing the diameter by 2

$$\text{radius} = 19 \div 2$$

[1]

**radius = 8.5 cm** [1]

Q6

Write down an expression for the angles around the point at which the triangle and the trapezium connect.

$$a + 65 + c + 115 = 360 \text{ (Angles around a point are equal to 360°)}$$

[1]

Simplify.

$$a + c = 180$$

[1]

Write down an expression for angles $b$ and $c$.

$$b + c = 180 \text{ (Co-interior angles sum to 180°)}$$

Equate the two angle expressions as both are equal to 180.

$$a + c = b + c$$

Subtract $c$ from both sides.

$\boldsymbol{a = b}$

*Both correct reasons given* [1]

Q7

Calculate the length of side QS on triangle QRS.
Use Pythagoras' theorem, $c^2 = a^2 + b^2$, where $a$ and $b$ are the lengths of the two shorter sides and $c$ is the length of the hypotenuse.

$$QS^2 = 4^2 + 8^2$$

$$QS = \sqrt{4^2 + 8^2}$$

$$QS = \sqrt{80}$$
$$QS = \sqrt{16 \times 5}$$
$$QS = 4\sqrt{5}$$

Now that you know the length QS you can use Pythagoras' theorem again to calculate the length QP on triangle PQS.

$$10^2 = QP^2 + \left(4\sqrt{5}\right)^2$$
$$10^2 - \left(4\sqrt{5}\right)^2 = QP^2$$
$$\sqrt{10^2 - \left(4\sqrt{5}\right)^2} = QP$$
$$\sqrt{100 - 80} = QP$$
$$\sqrt{20} = QP$$
$$2\sqrt{5} = QP$$

Compare the ratios of the lengths of two pairs of corresponding sides.

$$\frac{QR}{QP} = \frac{RS}{QS}$$
$$\frac{4}{2\sqrt{5}} = \frac{8}{4\sqrt{5}}$$

Cancel common factors in each fraction.

$$\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}}$$

These two pairs of corresponding sides have the same ratio, compare this ratio to the third pair of corresponding sides.

$$\frac{QR}{QP} = \frac{QS}{PS}$$
$$\frac{2}{\sqrt{5}} = \frac{4\sqrt{5}}{10}$$

Simplify the fraction on the right hand side.

$$\frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

Rationalise the denominator of the first fraction by multiplying by $\frac{\sqrt{5}}{\sqrt{5}}$.

$$\frac{2\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

Simplify.

$$\frac{2\sqrt{5}}{5} = \frac{2\sqrt{5}}{5}$$

[1]

**Triangles QRS and PQS are similar because they have three pairs of corresponding sides with the same ratio** [1]

Q8

Add the information given in the question to the diagram as below.

We know that triangle $APQ$ is similar to triangle $ABC$, because $\angle APQ = \angle ABC$ (corresponding angles), $\angle AQP = \angle ACB$ (corresponding angles) and $\angle PAQ = \angle BAC$ (shared angle). Therefore

$$\frac{AP}{AB} = \frac{PQ}{BC}$$

Replace the sides with the known lengths

$$\frac{AP}{9} = \frac{8}{10}$$

[1]

Multiply both sides by 9 to solve for $AP$

$$AP = \frac{8}{10} \times 9 = 7.2$$

**7.2 cm** [1]